FUNDAMENTAL EVEN- AND ODD-MODE WAVES FOR NONSYMMETRICAL COUPLED LINES IN NON-HOMOGENEOUS MEDIA

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Abstract

Propagation experiments show the even-mode to have equal voltage-magnitudes and the odd-mode equal current-magnitudes. A new even-mode/odd-mode analysis is thus given for nonsymmetrical coupled-lines in non-homogeneous media, leading to the most general formulation of the 4 x 4 Y-matrix.

Experimental Results

Recently a series of basic propagation experiments was conceived and performed in our laboratory by using the coupled line test fixture shown in Figure 1.

This consists of a precision-machined brass box, open at two opposite ends and having parallel ground planes at either .4" or .282" mutual distance; a 3" long substrate of various dielectric materials can be suspended along the median axis of the box, in a plane orthogonal to the ground planes and carries etched, broadside-coupled gold runs of unequal width, following the cross-section geometry shown in Figure 2. Single or dual wave-launchers can be fitted at either end of the suspended substrate to realize a number of two-port, three-port or full four-port configurations.

In this situation the even- and odd-mode waves are known to propagate at two different and generally constant velocities v_e and v_o [1]; thus the coupled-lines system may be said to be "asynchronous".

In the first out of the most directly relevant series of experiments an even-mode wave, with equal voltage magnitudes, was launched down nonsymmetrical, asychronous, broadside-coupled striplines etched upon a .007" alumina substrate by connecting both input ends of the coupled lines to the same 25 ps-risetime tunnel-diode step-generator, through a single 50 ohm coaxial line and wave-launcher. By using wide band samplers as sensors, the even-mode step waveform was found to propagate along both coupled lines at a common even-mode velocity, slightly lower than the velocity of light in vacuum, and to reappear synchronously at the output ends of the two lines, (see Figure 3), regardless of line asymmetry (.05" and .01" strip widths were used).

In a second experiment, dual launchers were fitted at either end of the 3" long alumina substrate and a push-pull 25 ps-risetime dual, push-pull tunnel diode step generator was connected at the input lines and adjusted for step-synchronism in an attempt to launch an odd-mode step-wave, with equal voltage magnitudes, down the nonsymmetrical pair of coupled lines. Both possible connections were tested, one having the positive input step on the wide line (low impedance line) and the negative step on the narrow line (high impedance line), the second connection being obtained by exchanging the step polarities between the lines. The results obtained with both connections (one is shown in Figure 4) proved the originally balanced, push-pull step-excitation, applied at the line inputs to break down along the line length, into two non-synchronous components consisting of a faster even-mode wave having equal voltage magnitudes on the two lines and the same polarity of the signal at the wide line input, and a much slower odd-mode wave having unequal voltage magnitudes and opposite polarities, with the largest voltage appearing across the narrow line.

This second component was then interpreted as an odd-mode wave with equal current magnitudes and opposite phases characterized by zero ground-return current.

In a third experiment, also using the 4 launcher arrangement, the push-pull input step-excitation was purposely unbalanced, by means of attenuators, while carefully maintaining synchronism between the two incoming step polarities. By tentatively adjusting the relative magnitude of the input steps, but always maintaining the largest signal at the input of the narrow line a situation was found in which the fast even-mode component could be considered as vanishing. Absolute cancellation of this component would obviously require a 50 ohm, wide-band, continuously adjustable vernier attenuation control which was not available at the time the experiments were performed.

The Physically Real Even- and Odd-mode Waves

As a consequence of these experimental results an hypothesis was formulated, postulating as only physical realities, for the general nonsymmetrical and asynchronous situations, as well as in the particular cases of symmetry and synchronism, an even-mode wave with equal voltage magnitudes and an odd-mode wave with equal current magnitudes on the two lines.[1] (Figure 5)

Re-interpretation of Earlier Results -- Nonsymmetrical Lines in Homogeneous Medium

In the light of this hypothesis, the even-mode wave with equal current magnitudes and the odd-mode wave with equal voltage magnitudes, which are known to propagate without breaking down into components along nonsymmetrical coupled lines embedded in an homogeneous medium [3] [4], can be interpreted as linear combinations of the above-postulated only physically real wave modes, blended in mutual proportions depending on the degree of line-to-line asymmetry. This linear superposition of wave-modes obviously propagates in an homogeneous medium as a single wave as a consequence of the equality of the mode-velocities ($v_e = v_o$). As a consequence the so-called even-mode impedances Z^a_{oe} and Z^b_{oe} as well as the so-called odd-mode admittances Y^a_{oo} and Y^b_{oo} can be interpreted as very specific terminations, matching the output-end of the coupled lines for both components of the very specific linear combinations of physically real wave-modes described above.

(1) This is essentially the same odd-mode postulated by Ekinge as generalization of the Reed and Wheeler method (see [2] page 86).

Derivation of the 4 x 4 Y-Matrix

The new hypothesis was then applied to the calculation of the 16 entries of the Y-matrix of the "Nonsymmetrical and Asynchronous Coupled-Line Four-Port". Y-parameter equations were written for the even-mode propagation and Z-parameter equations were written for the odd-mode propagation. The odd-mode equations were then converted into equivalent Y-parameter equations by parameter conversion formulas. Finally the currents at the four ports were computed, by linear superposition of the two modes, yielding each of the four currents as functions of the voltages at the four ports.

The resulting expressions of the 16 coefficients of these four equations, representing the 16 entries of the desired Y-matrix, respect all the mutual identities dictated by the line end-to-end symmetry and by the requirement of reciprocity between any pair of distinct ports, for any values of v_e and v_o.

The resulting 4 x 4 Y-matrix turns out to contain only six different entries, located according to the same pattern of the Y-matrix given by Cristal [4] for the nonsymmetrical synchronous case.

These 6 entries and their location in the Y-matrix are given by:

$$Y_{11} = Y_{22} = -\frac{j}{1 + R_3}\left(\frac{Y^a_{oe} R_3}{\tan \theta_e} + \frac{1}{Z^a_{oo} \tan \theta_o}\right) \qquad (1)$$

$$Y_{33} = Y_{44} = -\frac{j}{1 + R_3}\left(\frac{Y^b_{oe}}{\tan \theta_e} + \frac{R_3}{Z^b_{oo} \tan \theta_o}\right) \qquad (2)$$

$$Y_{12} = Y_{21} = \frac{j}{1 + R_3}\left(\frac{Y^a_{oe} R_3}{\sin \theta_e} + \frac{1}{Z^a_{oo} \sin \theta_o}\right) \qquad (3)$$

$$Y_{34} = Y_{43} = \frac{j}{1 + R_3}\left(\frac{Y^b_{oe}}{\sin \theta_e} + \frac{R_3}{Z^b_{oo} \sin \theta_o}\right) \qquad (4)$$

$$Y_{13} = Y_{31} = Y_{24} = Y_{42} = $$

$$= -\frac{j}{1 + R_3}\left(\frac{Y^a_{oe}}{\tan \theta_e} - \frac{1}{Z^a_{oo} \tan \theta_o}\right) =$$

$$= -\frac{j R_3}{1 + R_3}\left(\frac{Y^b_{oe}}{\tan \theta_e} - \frac{1}{Z^b_{oo} \tan \theta_o}\right) \qquad (5)$$

$$Y_{14} = Y_{41} = Y_{23} = Y_{32} = $$

$$= \frac{j}{1 + R_3}\left(\frac{Y^a_{oe}}{\sin \theta_e} - \frac{1}{Z^a_{oo} \sin \theta_o}\right) =$$

$$= \frac{j R_3}{1 + R_3}\left(\frac{Y^b_{oe}}{\sin \theta_e} - \frac{1}{Z^b_{oo} \sin \theta_o}\right) \qquad (6)$$

where:

$$Y^a_{oe} = v_e C_a \qquad (7)$$

$$Z^a_{oo} = \frac{C_b}{v_o\left(C_a C_b + C_a C_{ab} + C_b C_{ab}\right)} \qquad (8)$$

$$Y^b_{oe} = v_e C_b \qquad (9)$$

$$Z^b_{oo} = \frac{C_a}{v_o\left(C_a C_b + C_a C_{ab} + C_b C_{ab}\right)} \qquad (10)$$

$$R_3 = \frac{C_a}{C_b} \qquad (11)$$

$$\theta_e = \frac{\omega}{v_e} \ell \qquad (12)$$

$$\theta_o = \frac{\omega}{v_o} \ell \qquad (13)$$

Here C_a, C_b and C_{ab} are the capacitances per unit length and θ_e, θ_o are the electrical lengths for the two modes.

The new generalized expression of the Y-matrix should prove of great value in all known applications because of the added degree of freedom represented by the mode-asynchronism.

This added flexibility is demonstrated by the possibility of designing single-section directional couplers and transmission-line transformers with bandwidths far in excess of an octave (Figure 6).

References

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2. R.B. Ekinge, "A New Method of Synthesizing Matched Broadband TEM-mode Three-Ports," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-19, No. 1, January 1971.

3. H. Ozaki and J. Ishii, "Synthesis of a Class of Strip-Line Filters," IRE Transactions on Circuit Theory, Vol. CT-5, No. 2, June 1958, pp. 104-109.

4. E.G. Cristal, "Coupled-Transmission-Line Directional Couplers with Coupled Lines of Unequal Characteristic Impedances," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-14, No. 7, July 1966, pp. 337-346.

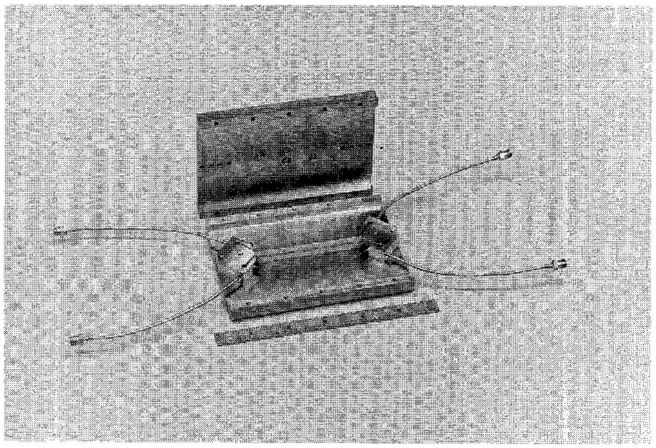

Fig. 1 Test fixture for fundamental odd-mode wave propagation experiment upon nonsymmetrical asynchronous parallel coupled lines. A 4-port launcher arrangement is shown. (Low impedance line shown in foreground.)

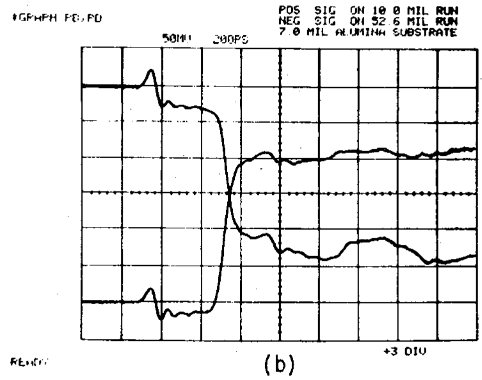

Fig. 4 Odd-mode step wave propagation experiment: The 25-ps risetime input steps having opposite polarity are seen to break down in a faster even-mode component having the polarity of the step at the input of the low impedance line and two slower odd-mode components having unequal voltage magnitude, the larger voltage appearing across the higher impedance line.

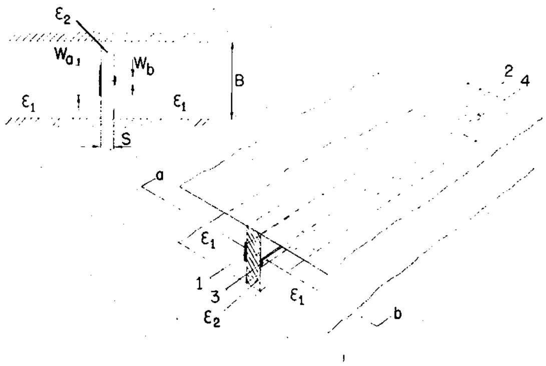

Fig. 2 Typical nonsymmetrical asynchronous coupled-line 4-port. Nonhomogeneous dielectric propagation medium is assumed. The cross-section geometry of the test fixture for fundamental wave-mode propagation experiment is shown in the insert.

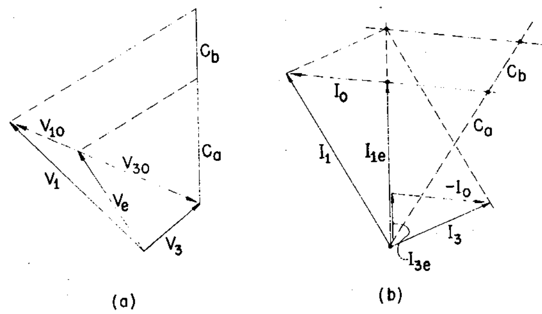

Fig. 5 In asymmetric coupled-line systems the decomposition of the wave voltages in even- and odd-mode components differs from the symmetric coupled-line situation because of the odd-mode components being unequal in magnitude and in reciprocal ratio of the capacitances per unit length of the two lines; the decomposition of the wave currents because of the even-mode components being unequal in magnitude and in the ratio of the capacitances per unit length of the two lines.

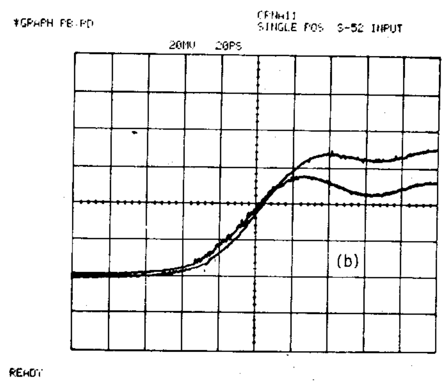

Fig. 3 Even-mode step wave propagation experiment. The two equal 25-ps risetime input steps are seen to reappear synchronously at the low impedance and the high impedance output ports.

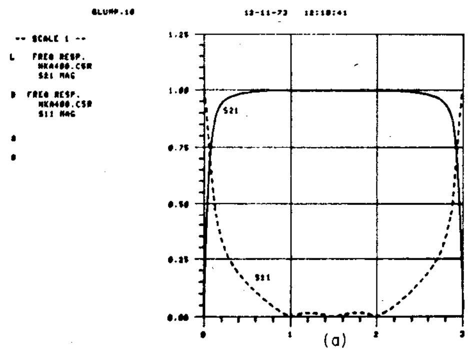

Fig. 6 Insertion gain $|S_{21}|$ and return loss $|S_{11}|$ of one typical asynchronous coupled-line transformers. Port 2 and 3 are grounded, while Port 1 and 4 are the input and output ports. The vertical scale is linear, the frequency scale also and expressed in GHz.